Q.1

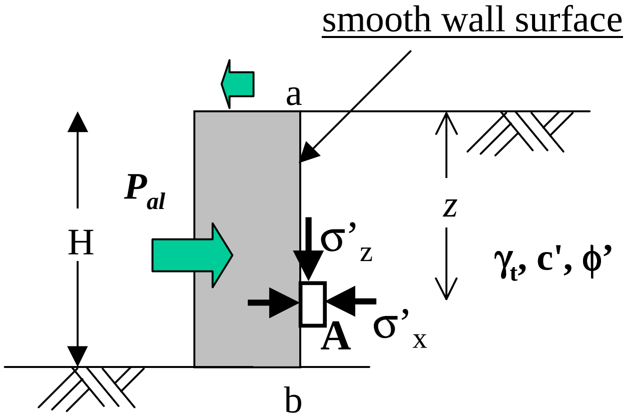

Fig.1

(1) As the shear stress on the vertical smooth wall surface is zero, the vertical and horizontal planes are principal stress planes.
For active conditions, $\sigma'_z < \sigma'_x$.
Hence,

$\sigma'_z = \gamma_t z = \sigma'_1$ (1)

and

$\sigma'_x = \sigma'_3$ (2)

This stress condition can be extended to the entire area satisfying the criteria as shown in Fig.A (Answer of q(1)).

(2) From the geometry in Fig.A,

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$$\frac{\sigma_z - \sigma_x}{2} = \sin\phi'\left(\frac{\sigma_z + \sigma_x}{2} + c\cot\phi'\right) \qquad (3)$$

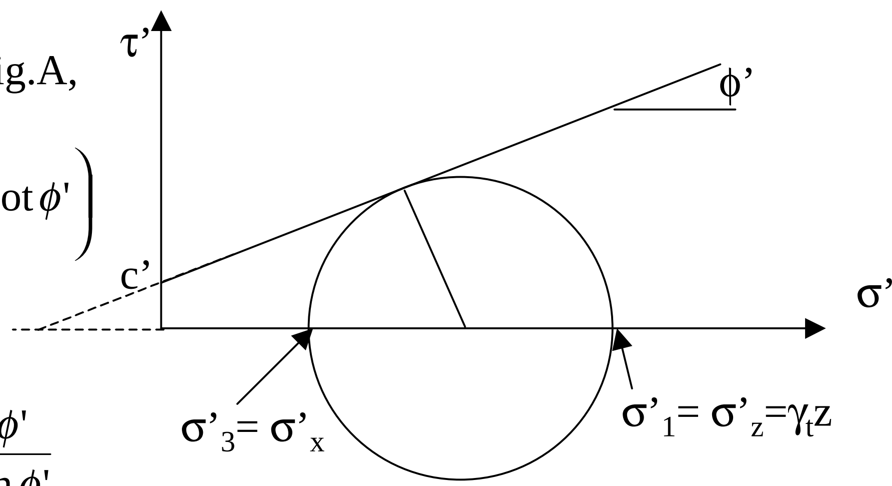


Fig.A

$$\sigma_x = \sigma'_z \frac{1-\sin\phi'}{1+\sin\phi'} - 2c'\frac{\cos\phi'}{1+\sin\phi'}$$

$$\sigma_x(z) = \gamma_t z \frac{1-\sin\phi'}{1+\sin\phi'} - 2c'\frac{\cos\phi'}{1+\sin\phi'} \qquad (4)$$

By integrating eq.(4) from to H, lower bound of active total earth pressure

$$\boxed{P_{al} = \gamma_t H^2 \frac{1-\sin\phi'}{2(1+\sin\phi')} - 2c'H\frac{\cos\phi'}{1+\sin\phi'}} \qquad (5)$$

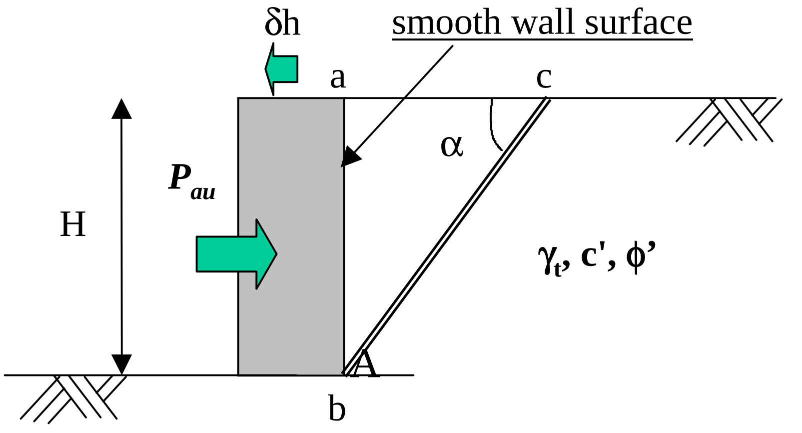


Fig.2

(3)

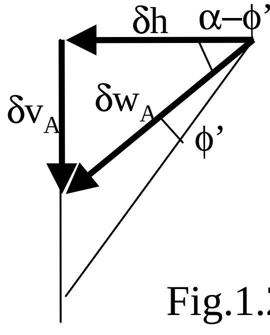


Fig.1.2

δw_A:displacement of block A
δv_A :Vertical displacement of block A

$$\delta w_A = \delta h / \cos(\alpha - \phi') \qquad (6)$$

$$\delta v_A = \delta h \tan(\alpha - \phi') \qquad (7)$$

(4)

$$\Delta V_A = \frac{1}{2} H^2 / \tan\alpha \qquad (8)$$

$$\Delta E = \delta h \cdot P_{au} + \delta v_A \cdot \gamma_t dV_A \qquad (9)$$

Hence δh and P_a direct oppositely, ΔE becomes as following using eqs. (7) and (8),

$$\Delta E = -\delta h P_{au} + \delta h \tan(\alpha - \phi') \frac{\gamma_t}{2} H^2 / \tan\alpha \qquad (10)$$

(5) Hence the wall surface is smooth, no energy dissipation occurs,

$$\Delta W = c' \cdot \overline{bc} \cdot \delta w_A \cos\phi' \qquad (11)$$

Using eq.(1) and $\overline{bc} = H / \sin\alpha$

$$\Delta W = c' \cdot \delta h H \frac{\cos\phi'}{\sin\alpha \cos(\alpha - \phi')} \qquad (12)$$

(6) From ΔE=ΔW and eqs. (10) and (12)

$$P_{au} = \frac{\gamma_t}{2} H^2 \frac{\tan(\alpha - \phi')}{\tan\alpha} - c' H \frac{\cos\phi'}{\sin\alpha \cos(\alpha - \phi')} \qquad (13)$$

(7)

$$\frac{dP_{au}}{d\alpha}=\frac{\gamma_t}{2}H^2\frac{\dfrac{\tan\alpha}{\cos^2(\alpha-\phi')}-\dfrac{\tan(\alpha-\phi')}{\cos^2\alpha}}{\tan^2\alpha}$$

$$-c'H\cos\phi'\frac{\cos\alpha\cos(\alpha-\phi')-\sin\alpha\sin(\alpha-\phi')}{\sin^2\alpha\cos^2(\alpha-\phi')}\qquad(14)$$

substituting $\alpha=45^o+\phi'/2$ into eq.(9), the following condition can be satisfied.

$$\frac{dP_{au}}{d\alpha}=\frac{\gamma_t}{2}H^2\frac{\dfrac{\tan(45^\circ+\phi'/2)}{\sin^2(45^\circ+\phi'/2)}-\dfrac{\cot(45^\circ+\phi'/2)}{\cos^2(45^\circ+\phi'/2)}}{\tan^2\alpha}\rightarrow 0$$

$$-c'H\cos\phi'\frac{\cos(45^\circ+\phi'/2)\cos(45^\circ-\phi'/2')-\sin(45^\circ+\phi'/2)\sin(45^\circ-\phi'/2')}{\sin^2\alpha\cos^2(\alpha-\phi')}\rightarrow 0\qquad(15)$$

$$=0$$

or

$$\frac{dP_{au}}{d\alpha}=\frac{\gamma_t}{2}H^2\frac{\sin\alpha\cos\alpha-\sin(\alpha-\phi')\cos(\alpha-\phi')}{\tan^2\alpha\cos^2(\alpha-\phi')\cos^2\alpha}$$

$$-c'H\cos\phi'\frac{\cos\phi'\cos2\alpha+\sin\alpha2\sin\phi'}{\sin^2\alpha\cos^2(\alpha-\phi')}=0\qquad(14')$$

$=0$
$\Rightarrow \sin\alpha=\cos(\alpha-\phi')\ \&\ \cos\alpha=\sin(\alpha-\phi')$
$\Rightarrow \alpha=45^o+\phi'/2$

$=0$
$\Rightarrow \tan 2\alpha=-\cot\phi'=\tan(90^o+\phi')$
$\alpha=45^o+\phi'/2$

substituting $\alpha=45^o+\phi'/2$ into eq.(13),

$$P_{au}=\frac{\gamma_t}{2}H^2\frac{\tan(45^\circ-\phi'/2)}{\tan(45^\circ+\phi'/2)}-c'H\frac{\cos\phi'}{\cos^2(45^\circ-\phi'/2)}$$

$$=\frac{\gamma_t}{2}H^2\frac{\sin^2(45^\circ-\phi'/2)}{\cos^2(45^\circ+\phi'/2)}-c'H\frac{2\cos\phi'}{1+\cos(90^\circ-\phi')}$$

$$=\frac{\gamma_t}{2}H^2\frac{1-\sin\phi'}{1+\sin\phi'}-2c'H\frac{\cos\phi'}{1+\sin\phi'}\qquad(16)$$

As $P_{al}=P_{au}$, eqs.(5) and (16) are exact solution of the active earth pressure on smooth vertical wall.

Q2. Explain the reason why the limit analysis can be reasonably applied to stability analyses on clay for short term problems and why it cannot be directly applied for the stability analyses on loose sand?

Applicability of limit analysis to the stability problem highly depends on how much the actual conditions are close to the conditions theoretically required in the analysis. That is the material should be perfect plastic material, following 1) non-plastic hardening type stress strain relation and 2) associate flow rule or normality rule.

As can be seen in Fig.2.1, dense sand does not show the hardening behavior after failure, while loose sand shows strain hardening at large strain level, which cannot satisfy condition 1).

For saturated clay in short term problem undrained condition can be reasonably assumed, because of low permeability, hence undrained strength c_u with ϕ_u=0 can be used as the failure criteria. As undrained condition in saturated soil secures zero volumetric strain, vector of plastic strain directs vertically, being normal to the failure criteria, satisfying associate flow rue as show in Fig.2.2(1).

For sand in which drained condition can be satisfied in normal practice, the failure criteria shown in Fig.2.2(2) is used. In order to satisfy the associate flow rule, the ratio of plastic volumetric strain to plastic shear strain, $-\delta\varepsilon^p_v/\delta\varepsilon^p_s$(=tanψ) should be tanϕ. This means quite large negative plastic volumetric strain $\delta\varepsilon^p_v$,should take place at failure. As shown in Fig.2.1(2), the volumetric strain increment for dense sand at failure is negative showing dilation, but for loose sand very small or even positive volumetric strain increment is normally observed. From these dilatancy (volume change characteristics due to shear), the associate flow rule can be reasonably satisfied for the dense sand but not for the loose sand.

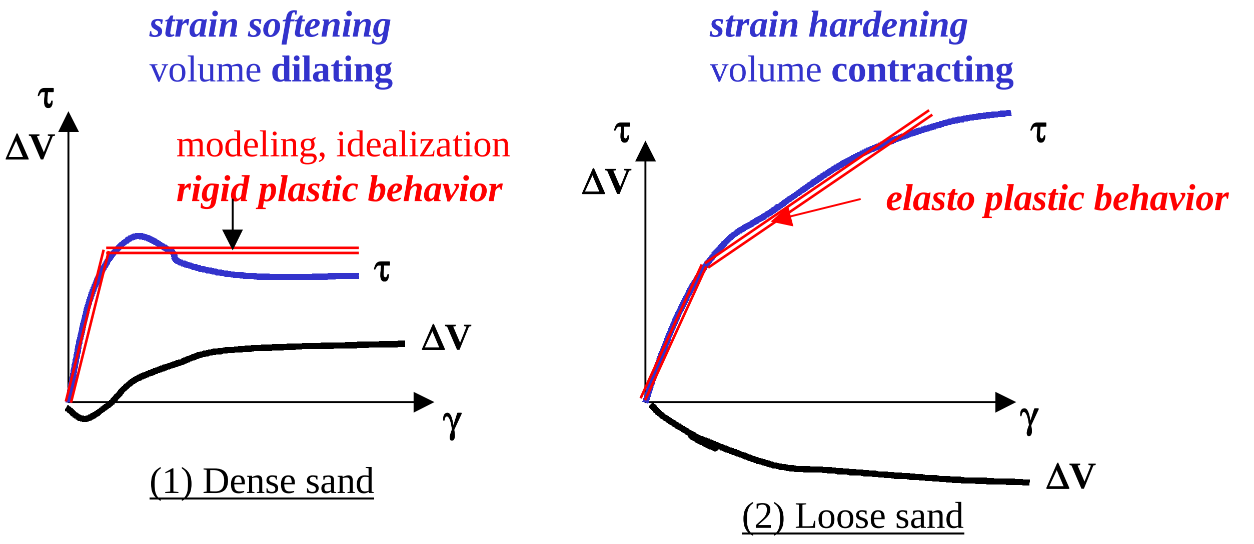


Fig.2.1 Typical shear stress – strain and Volume change –strain behavior

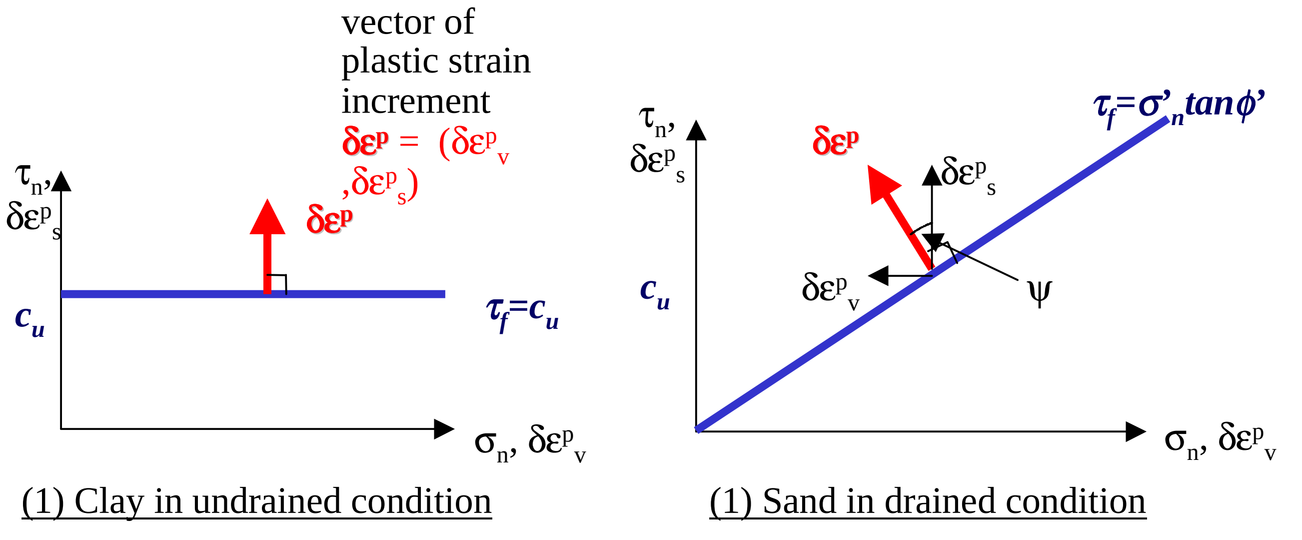


Fig.2.2 Failure criteria for clay in undrained condition ($\phi u=0$) and sand in drained condition

Q.3

(1)

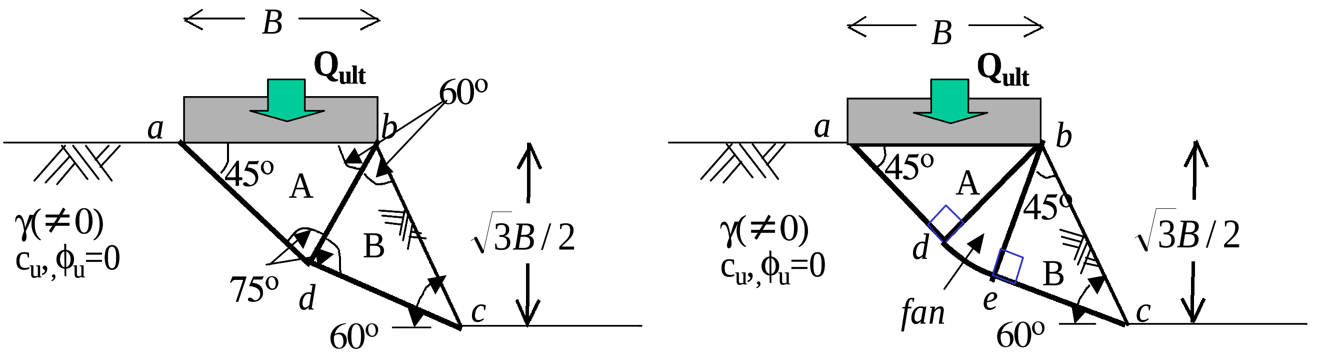


Fig.3

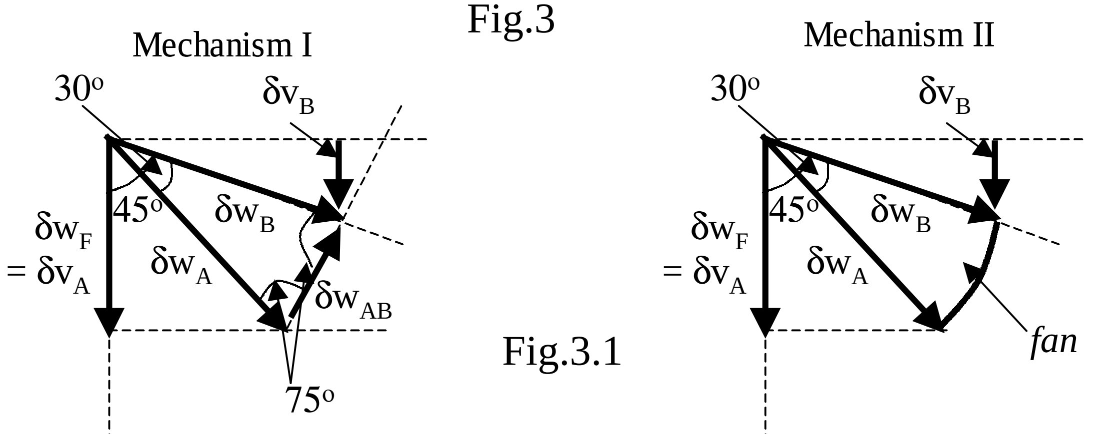


Fig.3.1

$$\delta v_A = \delta w_F,\quad \delta v_B = \delta w_B \sin 15^\circ \quad and \quad \delta w_B = \delta w_A = \delta w_F / \cos 45^\circ \qquad (1)$$

Table 3.1 Mechanism I

slip plane	shear stress	length L	displacement δw	$\delta W = c_u L \delta w$
ad	c_u	Bsin60°/sin75°	$\delta w_F/\cos45^\circ$	$\dfrac{Bc_u\delta w_F \sin60^\circ}{\cos45^\circ \sin75^\circ}$
db	c_u	Bsin45°/sin75°	$\dfrac{\delta w_F \sin30^\circ}{\cos45^\circ \sin75^\circ}$	$\dfrac{Bc_u\delta w_F \sin30^\circ}{\sin^2 75^\circ}$
dc	c_u	Bsin60°/sin75°	$\delta w_F/\cos45^\circ$	$\dfrac{Bc_u\delta w_F \sin60^\circ}{\cos45^\circ \sin75^\circ}$

Table 23.2 Mechanism II

slip plane	shear stress	length L	displacement δw	$\delta W = c_u L \delta w$
ad	c_u	Bcos45°	$\delta w_F/\cos45^\circ$	$Bc_u\delta w_F$
Fan	c_u	R=Bcos45° $\delta\theta=\pi/6$	$\delta w_F/\cos45^\circ$	$\dfrac{Bc_u\pi\delta w_F}{3}$
dc	c_u	Bcos45°	$\delta w_F/\cos45^\circ$	$Bc_u\delta w_F$

Volume of block A and B:

For Mechanism I: $$\delta V_A = \delta V_B = \frac{\gamma B^2 \sin 60^o \sin 45^o}{2\sin 75^o} \quad (2.I)$$

For Mechanism II: $$\delta V_A = \delta V_B = \frac{\gamma B^2}{4} \quad (2.II)$$

(2-I) External work of Mechanism I is given by Eq.(3.I)

$$\Delta E_I = \delta w_F \cdot Q_{ult} + \delta v_A \cdot \gamma dV_A + \delta v_B \cdot \gamma dV_B \quad (3.I)$$

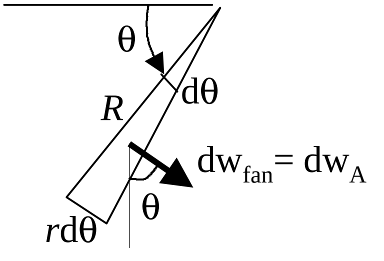
by substituting eqs.(1) and (2.I) in to eq.(3.I)

$$\Delta E_I = \delta w_F Q_{ult} + \frac{\delta w_F \gamma B^2}{2}\frac{\sin 60^o \sin 45^o}{\sin 75^o} + \frac{\delta w_F \gamma B^2}{2}\frac{\sin 60^o \sin 15^o}{\sin 75^o} \quad (4-I)$$

Since $\sin 75^o = \frac{\sqrt{6}+\sqrt{2}}{4}, \quad \sin 15^o = \frac{\sqrt{6}-\sqrt{2}}{4}$ (5)

$$\boxed{\Delta E_I = \delta w_F Q_{ult} + \frac{\delta w_F \gamma B^2}{2}\left(\frac{\sqrt{3}\sqrt{2}}{(\sqrt{6}+\sqrt{2})} - \frac{\sqrt{3}(\sqrt{6}-\sqrt{2})}{2(\sqrt{6}+\sqrt{2})}\right) = \delta w_F Q_{ult} + \frac{\sqrt{3}\delta w_F \gamma B^2}{4} \quad (6-I)}$$

(2-II) External work of Mechanism II is given by Eq.(3.II)



$$\Delta E_{II} = \delta w_F \cdot Q_{ult} + \delta v_A \cdot \gamma dV_A + \delta v_B \cdot \gamma dV_B + \delta E_{fan} \quad (3.II)$$

From the right figure, eq.(1), and Table 2

$$\delta E_{fan} = \frac{\gamma R^2 \delta w_A}{2}\int_{45^o}^{75^o}\cos\theta d\theta = \frac{\gamma B^2 \delta w_F \cos^2 45^o}{2\cos 45^o}\int_{45^o}^{75^o}\cos\theta d\theta = \frac{\gamma B^2 \delta w_F \cos 45^o}{2}\left(\sin 75^o - \sin 45^o\right) \quad (7)$$

by substituting eqs.(1) , (2.II) and (5) into eq.(3.II)

$$\Delta E_{II} = \delta w_F \cdot Q_{ult} + \frac{\delta w_F \gamma B^2}{4} + \frac{\delta w_F \gamma B^2}{4}\frac{\sin 15^o}{\cos 45^o} + \frac{\gamma B^2 \cos 45^o}{2}\left(\sin 75^o - \sin 45^o\right) \quad (4.II)$$

Using eq.(5)

$$\boxed{\Delta E_{II} = \delta w_F \cdot Q_{ult} + \frac{\delta w_F \gamma B^2}{4}\left(1 + \frac{\sqrt{6}-\sqrt{2}}{2\sqrt{2}} + \frac{\sqrt{2}(\sqrt{6}+\sqrt{2})}{4} - 1\right) = \delta w_F \cdot Q_{ult} + \frac{\sqrt{3}\delta w_F \gamma B^2}{4} \quad (6.II)}$$

(3-I) From Table 3.1, internal energy dissipation of Mechanism I is

$$\Delta W_I = 2Bc_u\delta w_F \frac{\sin 60^o}{\cos 45^o \sin 75^o} + Bc_u\delta w_F \frac{\sin 30^o}{\sin^2 75^o}$$

$$\Delta W_I = 2Bc_u\delta w_F \left(\frac{4\sqrt{3}}{\sqrt{2}\left(\sqrt{6}+\sqrt{2}\right)}\right) + Bc_u\delta w_F \frac{8}{\left(\sqrt{6}+\sqrt{2}\right)^2}$$

$$= 2Bc_u\delta w_F \left(\frac{2\sqrt{3}}{\sqrt{3}+1}\right) + Bc_u\delta w_F \frac{2}{\left(2+\sqrt{3}\right)^2}$$

$$= 2Bc_u\delta w_F \left(3-\sqrt{3}\right) + Bc_u\delta w_F \left(4-2\sqrt{3}\right)$$

$$\boxed{\Delta W_I = \left(10-4\sqrt{3}\right)2Bc_u\delta w_F \qquad (8-I)}$$

(3-II) From Table 3.2, internal energy dissipation of Mechanism II is

$$\boxed{\Delta W_{II} = 2Bc_u\delta w_F + \frac{\pi}{3} Bc_u\delta w_F \qquad (8.II)}$$

(4-I) From ΔE=ΔW and Eqs.(6.I) and (8.I),

$$\boxed{Q_{ultI} = \left(10-4\sqrt{3}\right)Bc_u - \frac{\sqrt{3}}{4}\gamma B^2 = \underline{3.07Bc_u} - \frac{\sqrt{3}}{4}\gamma B^2 \qquad (9.I)}$$

(4-II) From ΔE=ΔW and Eqs.(6.II) and (8.II),

$$\boxed{Q_{ultII} = \left(2+\frac{\pi}{3}\right)Bc_u - \frac{\sqrt{3}}{4}\gamma B^2 = \underline{3.047Bc_u} - \frac{\sqrt{3}}{4}\gamma B^2 \prec Q_{ultI} \qquad (9.II)}$$

(5)
Since the major principal stresses (σ_1(x)) direct vertically at the embankment top surface, the Mohr stress circle, G, at the point *g* and its pole are given as in Fig.2.4. From the figure, <u>the shear stress acting on the plane parallel to the line *gi* is **negative**</u>. Hence the slip line *gih* is <u>β slip line (discontinuity)</u> .

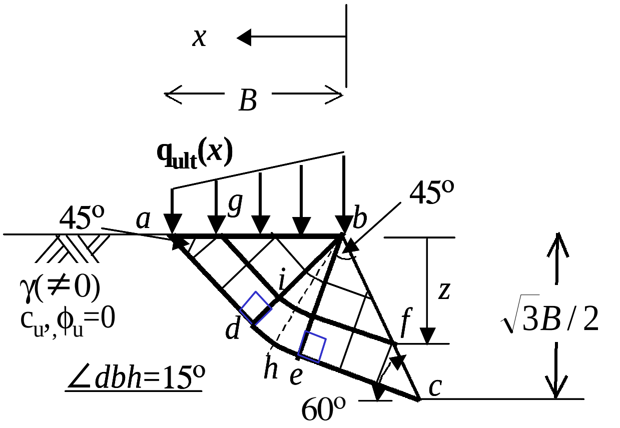


Fig.4 Slip line network

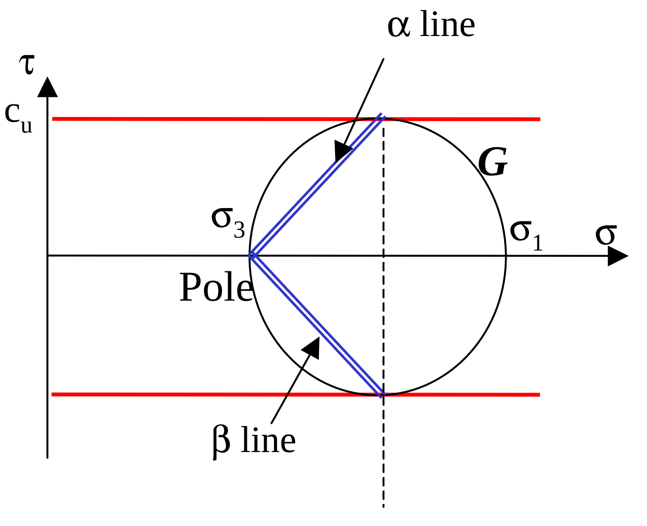


Fig.3.4

(6) Since the stresses normal and parallel to the slope are minor principal stress (σ_3=0) and major principal stresses respectively, the Mohr stress circle of the point *c* can be drawn as shown in Fig.2.5. And mean stress of point c is

$$s_c = c_u \qquad (9)$$

Change of mean stress Δs caused by rotation of major principal stress, Δη, and elevation change, Δz, along a β slip line for undrained conditions is given by

$$\Delta s = -2\Delta\eta c_u + \gamma\Delta z \qquad (10)$$

Since Δη=-15° (=-π/12) and Δz=-Bcos45° (sin15°+sin75°-sin60°) from point *c* to point *h* and Δη=-30° (=-π/6) and Δz=-Bsin60° from point *c* to point *h*,

$$\Delta s_{c\to h} = s_h - s_c = \frac{\pi}{6}c_u - 0.254B \quad \Rightarrow \underline{s_h = \frac{7\pi}{6}c_u - 0.254\gamma B \quad (11)}$$

$$\Delta s_{c\to h} = s_a - s_c = \frac{\pi}{3}c_u - \frac{\sqrt{3}}{2}B \quad \Rightarrow \underline{s_h = \frac{4\pi}{3}c_u - \frac{\sqrt{3}}{2}\gamma B \quad (12)}$$

Hence, Mohr circles of points *c*, *h* and *a* can be drawn as shown in Fig. 2.5.

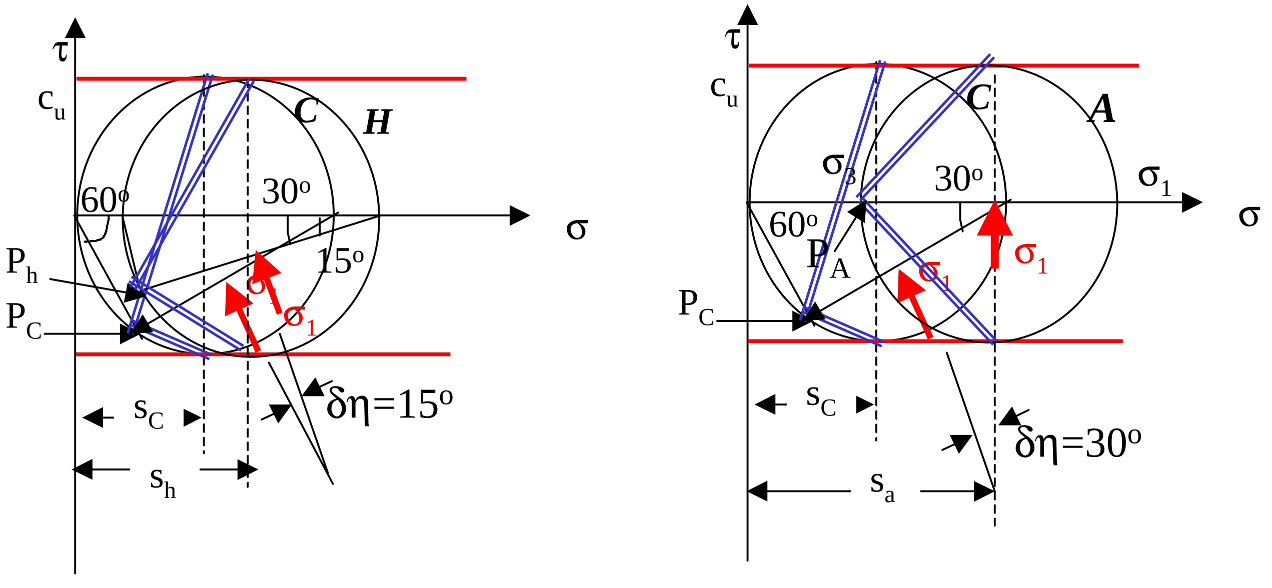


Fig.3.5

(7) Similar to (6), the change of mean stress from point *f* to g is obtained by eq.(10
and

$$\Delta s_{f\to g} = s_g - s_f = \frac{\pi}{3}c_u - \gamma\Delta z \;\Rightarrow s_g = \frac{4\pi}{3}c_u - \lambda\Delta z \longleftarrow \Delta\eta = -30^{\circ} = -\frac{\pi}{6}$$

$$q_{ult}(x) = s_g + c_u = \left(2+\frac{\pi}{3}\right)c_u - \frac{\sqrt{3}}{2}\gamma x \qquad (13) \longleftarrow \Delta z = \frac{\sqrt{3}}{2}x$$

Eq.(14) is Q_{ult} from the slip line method obtained by integrating eq.(13) from x=0 to B. It is equal to Q_{ultII} (eq.(8.II)) from the upper bound analysis using mechanism II.

$$Q_{ult} = \int_0^B q_{ult}(x)dx = \left(2+\frac{\pi}{3}\right)Bc_u - \frac{\sqrt{3}}{4}B^2 \qquad (14)$$